

Lattice QCD with 12 Quark Flavors: A Careful Scrutiny

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With a substantial amount of simulations, we have explored the system across a wide range of lattice scales. We have located a lattice artifact, first order bulk transition, have studied its properties, and found that the flavor-singlet scalar meson mass vanishes at the critical endpoint. We will discuss the lattice phase diagrams and the continuum limits for both a $S\chi SB$ phase and an IR conformal phase, and compare results with other groups.

Keywords: Lattice QCD; Many-flavor-QCD; Walking theory; Conformal theory

1. Introduction

It is both phenomenologically and theoretically interesting to study strongly coupled gauge theories with the number of massless fermions, N_f , tuned close to the lower limit of the conformal window, where the theory remains asymptotically free at high energies while developing a non-trivial infrared (IR) fixed point. While the system with N_f close to the upper limit of the conformal window can be studied perturbatively,¹ for systems with N_f close to the lower limit, the behavior is less clear due to the non-perturbative nature of gauge theories.

We have been studying the $SU(3)$ gauge theory with 8 and 12 fundamental fermions on the lattice, using the naive staggered fermion action and the DBW2 gauge action. We have explored the system through basic low-energy QCD observables with multiple volumes. In this paper, we will concentrate on our recent findings of the lattice artifact bulk transition with 12 flavors, discuss the continuum limit, and compare results with other groups. We quote results from largest volume available at each set of parameters, with finite volume effects of about a few percent.

2. Lattice critical point

We have located a line of first order bulk transitions² in the bare lattice parameter space: quark mass, m_q , and coupling, $\beta = 6/g^2$. This line of first order transitions

appears at relatively strong lattice couplings $\beta \sim 0.46$ and small input quark masses $m_q \lesssim 0.008$. Discontinuities in hadronic observables at the first order transition become smaller, as the quark mass, m_q , is increased. The system shows a quick cross-over region,³ at $m_q \geq 0.01$, with a marked change in lattice scales, from strong couplings to weaker couplings.

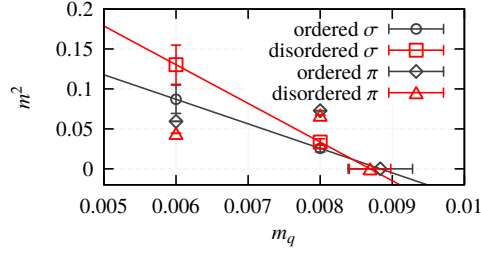


Fig. 1. The critical endpoint from extrapolating the mass of the flavor-singlet scalar meson

The system is approaching a second order transition, or a critical endpoint, as the quark mass, m_q , is increased while the lattice coupling, β , is tuned toward the end of the first order bulk transition. We have measured the mass of the flavor-singlet scalar meson, σ , using metastable ensembles at two sets of input parameters, detailed in Ref. 2. The mass of the flavor-singlet scalar meson, m_σ , depicted in Fig. 1, decreases on the line of the first order bulk transition, as m_q increases, and becomes even smaller than pion. To fit the critical behavior, we have used the mean-field scaling law, $m_\sigma \propto (m_q - m_q^C)^{1/2}$, which happens to describe our data quite well. The fitted location of the endpoint from ordered-start configurations (weak coupling) agrees with the fitted result from disordered-start configurations (strong coupling).

This critical behavior of m_σ , resembles the behavior found in finite temperature transitions.⁴ However, our findings with 12 quark flavors at zero temperature occur at finite m_q and β . While the continuum physics dictated by asymptotic freedom, can only be extracted at the limit of $m_q \rightarrow 0$ and $\beta \rightarrow \infty$, this critical point certainly does not have such a conventional continuum limit, and is irrelevant to the continuum physics we are after. Nonetheless, this critical point does possess a continuum limit, since the σ correlation length diverges at this point, while all other hadronic scales decouple from the theory. This second order endpoint thus gives a scalar field theory, and is likely non-interacting.^a

The first order bulk transition and critical endpoint² almost surely depends on the lattice action, as a few other groups have found bulk behaviors⁵⁻⁷ with different lattice actions and bare parameters. In the parameter space we have simulated, however, we did not observe the same characteristics of the bulk transition those groups advertised. We acknowledge that certain kind of lattice artifacts are highly

^aWe don't have hard proof, but it is hard to construct a nontrivial scalar field theory.

likely to exist in different lattice actions under different disguise and with different degrees of severity. Our findings of the critical behavior of the flavor-singlet scalar meson suggest that caution must be exercised in interpreting results from the scalar channel on the lattice, especially when asymptotic scaling is not well established.

3. Continuum limit

The continuum limit of lattice gauge theories can be analyzed by the renormalization group equation of any physical observable, $P(a, g, \hat{m})$, which depends on lattice spacing, a , bare lattice coupling,^b g , and bare lattice input quark mass, $\hat{m} = ma$. Near the continuum limit, P should mildly depend on a ,

$$a \frac{d}{da} P = \left\{ a \frac{\partial}{\partial a} - \beta(g, \hat{m}) \frac{\partial}{\partial g} - \gamma(g, \hat{m}) \frac{\partial}{\partial \hat{m}} \right\} P(a, g, \hat{m}) = \mathcal{O}(a), \quad (1)$$

where $\beta(g, \hat{m})$ and $\gamma(g, \hat{m})$ are lattice version of the renormalization group beta functions, and the right hand side of the equation is the usual scaling error.^c In an asymptotically free theory, to find the lattice beta functions, one would perturbatively compute some observable P , and plug the perturbative form back to Eq. (1). See Ref. 8 for a massless derivation using the quark potential. One will find the beta functions coincide with the continuum ones up to the lowest universal orders.^d In an asymptotically free theory, this procedure is self-consistent, because $g \rightarrow 0$, together with a fine-tuned $\hat{m} \rightarrow 0$, leads to $a \rightarrow 0$ according to the beta function calculated from this procedure.

Away from the physical continuum limit, one can supposedly use a form of the observable P that contains lattice higher order operators, say $P \rightarrow P + P_l$, where P_l is irrelevant in the usual continuum limit of physical QCD. With this additional term P_l , solving Eq. (1) has the possibility to lead to lattice beta functions, $\beta_l(g, \hat{m})$ and $\gamma_l(g, \hat{m})$, that are completely different from the usual continuum QCD, and contain lattice artifacts. One can solve these new lattice beta functions for $a \rightarrow 0$, and obtain a new continuum limit at $g \rightarrow g_C$ and $\hat{m} \rightarrow \hat{m}^C$, which is different from the usual continuum limit of physical QCD at $g \rightarrow 0$ and $\hat{m} \rightarrow 0$. As long as the result of observable $P + P_l$ is consistent with calculations in the limit of $g \rightarrow g_C$ and $\hat{m} \rightarrow \hat{m}^C$, such that Eq. (1) is self-consistent, we arrive at a lattice artifact continuum limit.

The second order critical endpoint we found with 12 flavors at finite lattice coupling β and input quark mass m_q , presented in the previous section, is such a lattice artifact continuum limit. The empirical lattice phase diagram is sketched in Fig. 2(a), where the axes are lattice bare parameters. We have two data points, which are located on the transition line and extended to smaller masses with a dashed line.

^bNot to be confused with renormalization beta functions, we use β and g interchangeably as the lattice bare coupling for ease of discussion.

^cThe size of the scaling violation depends on the lattice action and observable.

^dMinus signs in Eq. (1) ensure this agreement.

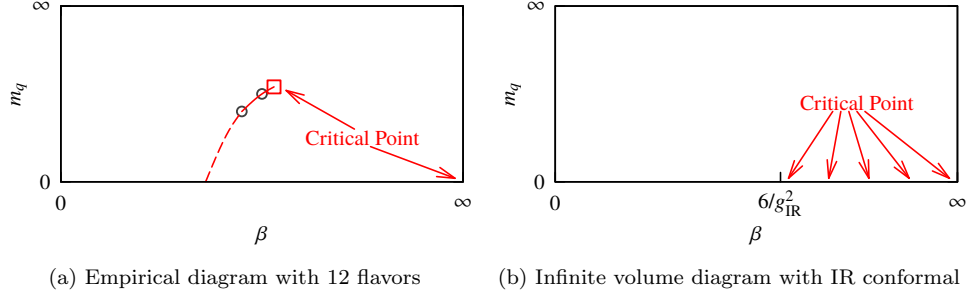


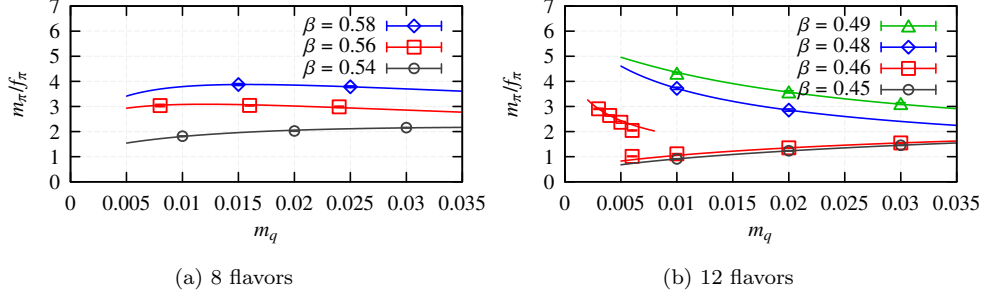
Fig. 2. Lattice phase diagrams

We note two critical points that correspond to the lattice artifact continuum limit and the physical QCD continuum limit, respectively.

To verify whether the 12-flavor theory is in the conformal window or remains in the spontaneously chiral symmetry breaking phase, one needs to know how the above picture of continuum limits would change if the underlying continuum theory has a non-trivial IR fixed point. In the following discussion, we assume the lattice is infinitely large, $V \rightarrow \infty$. Since the weak coupling limit of the theory is asymptotically free, the limit with $\beta \rightarrow \infty$ and $m_q \rightarrow 0$ is a valid continuum limit representing the true IR conformal continuum theory. It is conceivable that any non-zero quark mass term would break the conformal symmetry and the renormalized theory would lose the IR fixed point. If we keep $m_q = 0$, and vary β away from the weak coupling limit, since the renormalized coupling remains the fixed-point value, g_{IR} , the physics governed by the IR fixed point does not change, and neither do the lattice beta functions. Thus the theory remains IR conformal with a vanishing lattice spacing, $a = 0$. In terms of Eq. (1), any low-energy observable P remains under control of the IR fixed point, and is independent of the bare coupling, g , such that increasing g from 0 does not need to be compensated by increasing lattice spacing, a . In this case, Eq. (1) is self-consistently satisfied with $a = 0$ for $\hat{m} = 0$ and $0 \leq g \leq g_{\text{IR}}$. In conclusion, shown in Fig. 2(b), in the infinite volume limit, any point along the line with $m_q = 0$ and $\beta \geq 6/g_{\text{IR}}^2$ is a lattice critical point, and shares the same continuum limit as the weak coupling limit, which describes the same conformal theory at low energies. On the other hand, if $m_q \neq 0$, the theory is no longer conformal, and lattice artifacts are present with $a \neq 0$, which can be controlled by the bare coupling, β . This proposed scenario appears consistent with the data and we are investigating it further.

4. Mass ratios and universality

The dimensionless quantity, m_π/f_π , is shown in Fig. 3 versus the bare input quark mass, for both 8 flavors (left panel) and 12 flavors (right panel), respectively. We have drawn simple polynomial fit curves on top of our data. The discontinuity at $m_q = 0.006$ and $\beta = 0.46$ with 12 flavors represents the bulk transition. No bulk

Fig. 3. Scaling of constant physics by m_π/f_π

transition has been observed within our simulation parameters with 8 flavors.

We observed that, at strong couplings, $\beta = 0.54$ and $\beta = 0.45$ for 8 and 12 flavors respectively, the value of m_π/f_π decreases as m_q decreases. This behavior is expected in a physical QCD simulation, where chiral symmetry is spontaneously broken at $m_q \rightarrow 0$, and m_π vanishes while f_π remains finite in the limit. On the contrary, at weaker couplings, $\beta \geq 0.56$ for 8 flavors, and $\beta \geq 0.46$ after the bulk transition for 12 flavors, the m_π/f_π increases as m_q decreases. With 8 flavors, m_π/f_π is almost constant, and the fitted curves start to bend towards zero at small m_q , while the fitted curves with 12 flavors continues to grow in the parameter range we have simulated. Despite this difference, at weak couplings, m_π/f_π with 12 flavors resembles that with 8 flavors at relatively large $m_q \gtrsim 0.015$.

If we look at the constant physics naively with fixed m_π/f_π at weak couplings of the 12-flavor simulations, in order to reach the massless quark limit, we have to decrease β and go to stronger couplings. This is in contrast with the continuum limit discussed in the last section, and thus we cannot proceed with this procedure. The reason for this could be: a) lattice artifacts; b) physical behaviors.

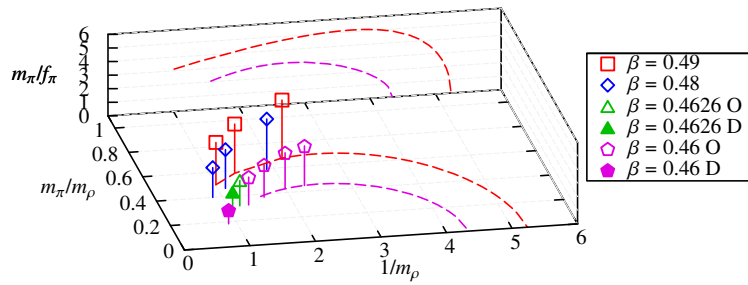


Fig. 4. Diagram of dimensionless ratios versus typical lattice correlation length

We have assembled values of both m_π/f_π and m_π/m_ρ from our 12-flavor simulations and plotted them against a typical lattice correlation length $1/m_\rho$ in Fig. 4. In the key of the figure, for ensembles with fixed β across both sides of the first

order bulk transition, weak coupling side (ordered-started) and strong coupling side (disordered-started) are labeled with O and D, respectively. The critical endpoint is located near the data points with $\beta = 0.4626$. In the figure, the length of the vertical line attached to each symbol represents m_π/f_π , and its position on the bottom layer shows the value of m_π/m_ρ versus the corresponding $1/m_\rho$. Dashed lines are polynomial extrapolations inspired by spontaneous-chiral-symmetry-breaking ($S\chi SB$) behaviors,

$$m_\pi^2 = c_0 m_q (1 + c_1 m_q), \quad (2)$$

$$m_\rho \text{ or } f_\pi = b_0 + b_1 m_q + b_2 m_q^2, \quad (3)$$

acted on each individual channel separately. Assuming $S\chi SB$ behaviors, $1/m_\rho$ needs to be increasingly larger, with the coupling weakening, to observe the ratios decrease as a result of vanishing m_π . On the other hand, if the theory does indeed have an IR fixed point, ratios in the infinite volume limit should approach constant values with different couplings, as $1/m_\rho \rightarrow \infty$, while all the dimensional quantities vanish in the conformal limit. Therefore, smaller quark masses would have stronger differentiating power to tell the true behavior of the system.

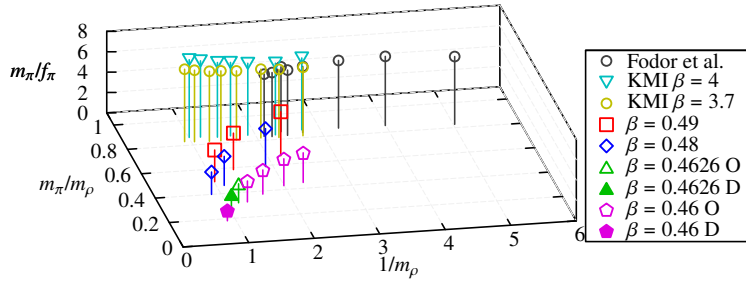


Fig. 5. Comparing dimensionless ratios with results from Fodor et al.⁹ and LatKMI¹⁰

To compare our results with results from other groups, we have included recent 12-flavor results^e from Fodor et al.⁹ and LatKMI,^{10f} in Figure 5. Although results of different groups are from different lattice actions, there is the universal behavior to be learned from the similar behaviors within the parameter space that has been explored. Apparently, both ratios, m_π/f_π and m_π/m_ρ are not changing noticeably as $1/m_\rho$ increases with fixed β within the parameter space limited by the computing resources. However, this behavior is not different from the behavior of m_π/f_π , shown in Fig. 3(a), for a system with 8 flavors. On the other hand, the ratios do seem to approach a universal continuum limit at $1/m_\rho \rightarrow \infty$, if the tendency continues without alteration in the region with small quark masses.

^eResults from the largest volume available are taken, and the finite volume effects are small.

^fValues of f_π from LatKMI are normalized by dividing $\sqrt{2}$.

5. Summary

We have located a lattice artifact, critical endpoint, where the scalar correlation length diverges, for 12-flavor QCD with naive staggered fermions and the DBW2 gauge action. In the weak coupling side of the lattice artifact bulk transition, we see confined, massive particle states for all simulations done to date. As the input quark mass decreases at a fixed lattice coupling, the hadronic scales (m_ρ and f_π) decrease much more rapidly than the pion mass, such that m_π/f_π or m_π/m_ρ increases on the weak coupling side of the bulk transition. The lattice input quark mass at the critical endpoint of the lattice artifact transition gives us a hint of the size of quark masses, at that lattice coupling, where the vanishingly small scalar mass can wildly distort continuum physics.

We have argued that an exact same continuum limit of an asymptotically free, IR conformal theory can be approached at the massless quark limit and the infinite volume limit, with any lattice bare coupling that is weaker than the renormalized coupling at the IR fixed point.

It may not be surprising that much lighter quark masses are required in lattice simulations for the $S\chi SB$ behavior to be numerically indisputable. Nevertheless, how to effectively scale towards the continuum limit within our current computational resources remains a question for both $S\chi SB$ and IR conformal scenarios.

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